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# Transmission Dynamics Model Analysis of HBV using Homotopy Perturbation Method

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***Abstract:****The transmission dynamics mathematical model of infectious disease is an essential disease controlling technique, which is being used on the occurrence of HBV to value the varying immunization strategies. In this paper, we analyze the transmission dynamics models through mathematical modeling using Homotopy perturbation method (HPM) which defines how to control the impact of HBV. To get the solution for nonlinear ordinary differential equations,Homotopy Perturbation Method (HPM) has been used. Here, we have discussed the numerical simulations up to six order approximation and error analysis with the help of Matlab software. SIDBRA model has been considered as the best modal to control the viral infections. Thus, examining the dynamics of Hepatitis B viral infection is mainly focused and also shows how given antibiotic (vaccination) control the disease.*
***Keywords:*** *Mathematical modeling, HPM, HBV, Dynamics, Transmission.*

## I. Introduction

Infection by Hepatics B virus is a threatening disease which may or may not show any symptoms to identify [1,3]. According to the statistical survey, it has been reported that more than 350 million people globally carriers Hepatitis B and die 0.6 million per year [1]. To protect and prohibit from this disease, timing vaccination is the possible solution to control the disease [6,13]. Particularly this has been proved by different research works.Mathematical modeling was a tool to be used for describing the complex process of HBV transmission. Differential equation models have been used in many of the published research studies [1]. Ordinary differential equation has been mainly used for mathematical expression compared to partial differential equation when the transmission dynamics of Hepatitis B modeling is done [1, 2].Age itself is the reasons for curing the populations of infectious diseases. If the person is at the young age, it is very easy to control [36]. Keeping the age as base, mathematical modelswas developed for studying the transmission dynamics of Hepatitis B. Zhao et al developed the following partial differential equation model with age structure [18]:

$$\begin{cases} \dfrac{\partial S(a,t)}{\partial a}+\dfrac{\partial S(a,t)}{\partial t}=-[\lambda(a,t)+v(a,t)+\mu(a)]S(a,t), \\ \dfrac{\partial I(a,t)}{\partial a}+\dfrac{\partial I(a,t)}{\partial t}=\lambda(a,t)S(a,t)-[v+\mu(a)]I(a,t), \\ \dfrac{\partial D(a,t)}{\partial a}+\dfrac{\partial D(a,t)}{\partial t}=vI(a,t)-[p(a)+\gamma+\mu(a)]D(a,t), \\ \dfrac{\partial B(a,t)}{\partial a}+\dfrac{\partial B(a,t)}{\partial t}=p(a)D(a,t)-[\delta(a)+\mu(a)+\mu_c(a)]B(a,t), \\ \dfrac{\partial R(a,t)}{\partial a}+\dfrac{\partial R(a,t)}{\partial t}=v(a,t)S(a,t)+\gamma D(a,t)+\delta(a)B(a,t)-\mu(a)R(a,t). \end{cases} \quad (1.1)$$

Here S, I, D, B, and R denote the proportion of individuals at the stage of sensitive, idle, dangerous, bearer, and recovery to HBV in the total population respectively. *a* is age and *t* is time. λ is the force of infection, v is the

rate of successful vaccination, natural mortality rate is μ, age dependent mortality is$\mu_c$ and the recovery rate is δ are assumed as constant.

The transmission dynamics model is recognized as compartmental model by which thevariation of HBV can be studied theoretically [1]. It is based on the characteristics of population and disease, dynamic behavior of disease using mathematical modeling.Zou, Zhang and Ruan proposed that how to control HBV using modeling the transmission dynamics [19, 20]. McLean and Blimberg first offered a HBV transmission differential equation model [16]. The impact of different antibiotic methods on the HBV is different. The antibiotic methods played a dominant role in newborns in subsidizing the effect of HBVfrequency, and immunization of sensitive adults or dangerous population have a reasonable effect on controlling HBV infection [21]. Therefore, the age and time are the most important factors to cure HBV infection.

Mathematical models have been one of the very much useful methods for the understanding virus and dynamics under treatment in infections such as HBV [22, 23].Sensitive S(t) denoted the population at precarious of infection with HBV; Idle I(t) denoted the population infected but not yet infectious; Dangerous D(t) denoted the population at early high infectious stage of HBV; Bearer B(t) denoted the population with continuing HBV infection who are infectious or non-infectious to others; Reacquire R(t) denoted the recovered population for the lifetime; Antibiotic A(t) denoted the immunity that monitors vaccination may disappear over time.

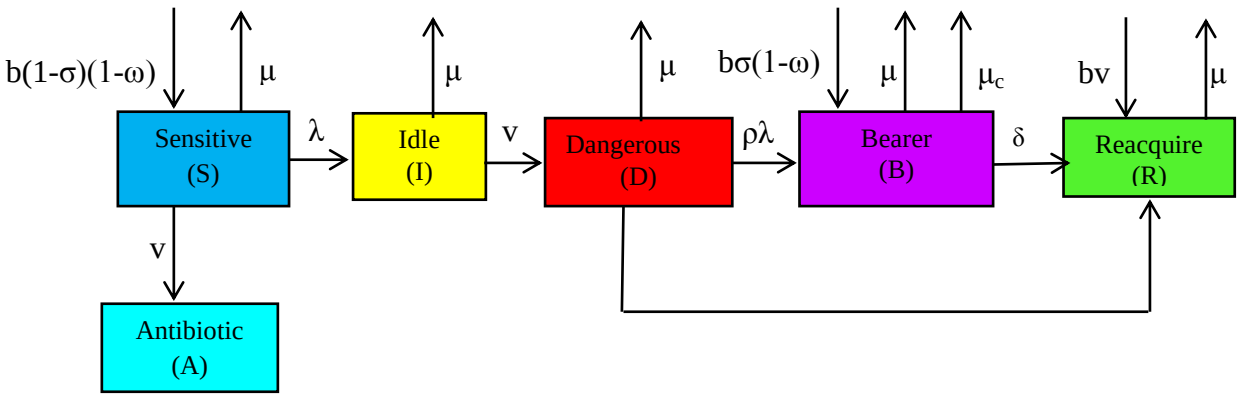



**Figure** 1: Compartmental diagram for the HBV.

Peifeng Liang et al [1] developed the following transmission dynamics models using six ordinary differential equations. Here they introduced one extra compartment which is vaccination (Antibiotic).The model becomes

$$\begin{cases} \dfrac{dS}{dt} = b - \omega B - b\sigma + b\sigma\omega B + \phi A - (\mu + \lambda + v)S \\ \dfrac{dI}{dt} = \lambda S - (\mu + v)I \\ \dfrac{dD}{dt} = vI - (\mu + \gamma)D \\ \dfrac{dB}{dt} = b\sigma(1-\omega)B + \rho\lambda D - (\mu + \mu_c + \delta)B \\ \dfrac{dR}{dt} = (1-\rho)\gamma D + \delta B - \mu R \\ \dfrac{dA}{dt} = b\omega + vS - (\phi + \mu)A \end{cases} \tag{1.2}$$

The initial and boundary conditions are:

$S(0) = 1\times10^{-1}$; $I(0) = 1\times10^{-2}$; $D(0) = 1\times10^{-3}$; $B(0) = 1\times10^{-4}$; $R(0) = 1\times10^{-5}$; $A(0) = 1\times10^{-6}$.

**TABLE 1.** Parameter values used in numerical simulations

| Parameter | Explanation | Range |
|---|---|---|
| μ | Natural mortality rate | 0.055 – 0.016 |
| λ | Force of HBV infection | 0.013 – 0.159 |
| ν | Rate of individuals leave the idle class | 6 – 8 per year |
| β | Coefficient of transmission | 0.8 – 20.49 |
| ω | The birth proportion with successful vaccination | 0.055 – 0.095 |
| σ | proportion of perinatal infection | 0.7 – 0.9 |
| δ | The recovery rate of bearer | 0.0005 – 0.03 |
| b | Birth rate | 0.0121 – 0.05 |
| ϕ | Rate of waning vaccine-induced immunity | 0.001 – 0.039 |
| γ | Rate of individuals leave the dangerous class | 3 – 4 per year |
| ρ | Probability of individual suffering from dangerous HBV infection to become a chronic bearer. | 0.05 – 0.09 |

## II. Homotopy Perturbation Method

To explain the primary concept of HPM, we take the non-linear functional equation as follows

$$P(x) - g(a) = 0\ ,\ a \in \Omega, \tag{2.1}$$

The boundary conditions are;

$$Q = \left( x, \frac{\partial x}{\partial n} \right) = 0\ ,\ a \in \Gamma\ , \tag{2.2}$$

Here P is an arbitrary functional operator, Q is a boundary operator, g(a) is ananalytic function $\Gamma$ and is the boundary of the domain $\Omega$. Generally, the operator P can be divided into two parts $T_L$ and $T_N$, where $T_L$ is a linear and $T_N$ is a non-linear operator. Therefore Eq (2.1) can be rewritten as follows;

$$T_L(u) + T_N(u) - g(a) = 0\,. \tag{2.3}$$

We construct a homotopy $h(a, p) : \Omega \times [0,1] \to R$ which satisfies

$$M(h, p) = (1-p)\left[T_L(h) - T_L(x_0)\right] + p\left[P(h) - g(a)\right] = 0, \tag{2.4}$$

Or

$$M(h, p) = T_L(h) - T_L(x_0) + p\ T_L(x_0) + p\left[T_N(h) - g(a)\right] = 0, \tag{2.5}$$

Where $p \in [0,1]$ is an embedding parameter, and $x_0$ is an initial approximation for the solution of Eq. (2.1), which satisfies the boundary conditions.By using HPM, let us use p, and take the solution of Eq. (2.5) should be written as a power series in p:

$$h = h_0 + h_1 p + h_2 p^2 + ... = \sum_{i=0}^{\infty} h_i p^i \tag{2.6}$$

Let us consider p =1, the approximate solution of Eq. (2.2) should be established as follows;

$$x = \lim_{p \to 1} h = h_0 + h_1 + h_2 + ... \tag{2.7}$$

**2.1 Convergence of the method**

Let's rewrite the Eq. (2.5) as the following;

$$T_L(v) - T_L(u_0) = p\left[g(a) - T_L(x_0) - T_N(h)\right] \tag{2.8}$$

Substituting (2.7) into (2.8) we get;

$$T_L\left(\sum_{i=0}^{\infty} h_i p^i\right) - T_L(x_0) = p\left[g(a) - T_L(x_0) - T_N \sum_{i=0}^{\infty} h_i p^i\right] \tag{2.9}$$

Therefore,

$$\sum_{i=0}^{\infty} T_L(h_i) - T_L(x_0) = p\left[g(a) - T_L(x_0) - T_N\left(\sum_{i=0}^{\infty} h_i p^i\right)\right] \tag{2.10}$$

By usingMaclaurin expansion of $T_N\left(\sum_{i=0}^{\infty} h_i p^i\right)$with respect to p, we get;

$$T_N\left(\sum_{i=0}^{\infty} h_i p^i\right) = \sum_{n=0}^{\infty}\left(\frac{1}{n!}\frac{\partial^n}{\partial p^n} T_N\left(\sum_{i=0}^{\infty} h_i p^i\right)\right)_{p=0} p^i \tag{2.11}$$

$$\left(\frac{\partial^n}{\partial p^n} T_N\left(\sum_{i=0}^{\infty} h_i p^i\right)\right)_{p=0} = \left(\frac{\partial^n}{\partial p^n} T_N\left(\sum_{i=0}^{\infty} h_i p^i\right)\right)_{p=0} \tag{2.12}$$

$$T_N\left(\sum_{i=0}^{\infty} h_i p^i\right) = \sum_{n=0}^{\infty}\left(\frac{1}{n!}\frac{\partial^n}{\partial p^n} T_N\left(\sum_{i=0}^{\infty} h_i p^i\right)\right)_{p=0} p^i \tag{2.13}$$

We assume,

$$M_n(h_0, h_1, h_2, ..., h_n) = \left(\frac{1}{n!}\frac{\partial^n}{\partial p^n} T_N\left(\sum_{i=0}^{\infty} h_i p^i\right)\right)_{p=0}, n = 0, 1, 2, ..., \tag{2.14}$$

Here $M_n$s are the so-called He's polynomials (2.11 – 2.13). Then,

$$T_N\left(\sum_{i=0}^{\infty} h_i p^i\right) = \sum_{n=0}^{\infty} M_i p^i \tag{2.15}$$

Substituting (2.15) into (2.10), we can derive;

$$\sum_{i=0}^{\infty} T_L(h_i) - T_L(x_0) = p\left[g(a) - T_L(x_0) - \sum_{n=0}^{\infty} M_i p^i\right] \tag{2.16}$$

By equating the terms with the identical powers in p:

$$\begin{cases} p^0 : T_L(h_0) - T_L(x_0) = 0 \\ p^1 : T_L(h_1) = g(a) - T_L(x_0) - M_0 \\ p^2 : T_L(h_2) = -M_1, \\ \vdots \\ p^n : T_L(h_{n+1}) = -M_n, \\ \vdots \end{cases} \tag{2.17}$$

So we can derive

$$\begin{cases} h_0 = x_0, \\ h_1 = T_L^{-1}[g(a)] - x_0 - T_L^{-1}(M_0), \\ h_2 = -T_L^{-1}(M_1), \\ \vdots \\ h_{n+1} = -T_L^{-1}(M_n), \\ \vdots \end{cases} \tag{2.18}$$

**Theorem2.1.***The solution of Eq. (2.1) is equivalent to defining the following using Homotopy perturbation method;*

$$k_n = h_1 + \ldots + h_n, \tag{2.19}$$

$$k_0 = 0,$$

By using the iterative system:

$$k_{n+1} = -T_L^{-1} T_{N_n}(k_n + h_0) - x_0 + T_L^{-1}(g(a)), \tag{2.20}$$

Here

$$T_{N_n}\left(\sum_{i=0}^{\infty} h_i\right) = \sum_{n=0}^{n} M_i, \quad \text{n=0, 1, 2, …}$$

(2.21)

**Proof.** For n=0, from (20), we have;

$$k_1 = -T_L^{-1} T_{N_0}(k_0 + h_0) - x_0 + T_L^{-1}(g(a)),$$
$$= -T_L^{-1}(M_0) - x_0 + T_L^{-1}(g(a)).$$

(2.22)

Then

$$h_1 = -T_L^{-1}(M_0) - x_0 + T_L^{-1}(g(a)).$$

(2.23)

For n=1:

$$k_2 = -T_L^{-1} T_{N_1}(k_1 + h_0) - x_0 + T_L^{-1}(g(a))$$
$$= -T_L^{-1}(M_0 + M_1) - x_0 + T_L^{-1}(g(a))$$
$$= -T_L^{-1}(M_1) + h_1.$$

(2.24) But $k_2 = h_1 + h_2$, we get;

$$h_2 = -T_L^{-1}(M_1).$$

(2.25)

Therefore the theorem will be proved. Weconsider $h_{r+1} = -T_L^{-1}(M_r)$, for r = 1, 2, …, n – 1,

$$k_{n+1} = -T_L^{-1} T_{N_n}(k_n + h_0) - x_0 + T_L^{-1}(g(a)),$$

$$= -T_L^{-1}\left(\sum_{n=0}^{n} M_i\right) - x_0 + T_L^{-1}(g(a)),$$

$$= -\left(\sum_{n=0}^{n} T_L^{-1}(M_i)\right) - x_0 + T_L^{-1}(g(a))$$

$$= h_1 + h_2 + \ldots + h_n - T_L^{-1}(M_n).$$

(2.26)

Then, from (2.19), it can be written as;

$$h_{n+1} = -T_L^{-1}(M_n).$$

(2.27)

Which is the result of (2.18) from HPM, and hence the theorem is proved.

**Theorem 2.2.**Let Ƀ be a Banach space.

a) $\sum_{i=0}^{\infty} h_i$ obtained by (18), convergence to k Є Ƀ, if Ǝ (0 ≤ γ < 1), s.t $(\forall n \in N \Rightarrow \| h_n \| \leq \gamma \| h_{n-1} \|)$.

(2.28)

b) $k = \sum_{n=1}^{\infty} h_n$, satisfies in $k = -T_L^{-1} T_N (k + h_0) - x_0 + T_L^{-1}(g(a))$.

(2.29)

**Proof.**

a) We have

$$\| k_{n+1} - k_n \| = \| h_{n+1} \| \leq \gamma \| h_n \| \leq \gamma^2 \| h_{n-1} \| \leq \ldots \leq \gamma^{n+1} \| h_0 \|.$$

(2.30)

For any n, m Є N, n ≥ m, we drive;

$$\begin{aligned}
\| k_n - k_m \| &= \| (k_n - k_{n-1}) + (k_{n-1} - k_{n-2}) + \ldots + (k_{m-1} - k_m) \|, \\
&\leq \| k_n - k_{n-1} \| + \| k_{n-1} - k_{n-2} \| + \ldots + \| k_{m-1} - k_m \|, \\
&\leq \gamma^n \| h_0 \| + \gamma^{n-1} \| h_0 \| + \ldots + \gamma^{m+1} \| h_0 \| \\
&\leq (\gamma^n + \gamma^{n-1} + \ldots + \gamma^{m+1}) \| h_0 \|, \\
&\leq (\gamma^{m+1} + \ldots + \gamma^n + \ldots) \| h_0 \|, \\
&\leq \gamma^{m+1} (1 + \gamma + \ldots + \gamma^n + \ldots) \| h_0 \|, \\
&\leq \frac{\gamma^{m+1}}{1-\gamma} \| h_0 \|.
\end{aligned}$$

(2.31)

So

$$\lim_{n,m \to \infty} \| k_n - k_m \| = 0$$

(2.32)

Then $\{k_n\}$, is Cauchy sequence in Banach space, and it is convergent, i.e.,

Ǝ k Є Ƀ, s.t $\lim_{n \to \infty} k_n = \sum_{n=1}^{\infty} h_n = k.$

(2.33)

b) From Eq. (2.30), we have;

$$\begin{aligned}
\lim_{x \to \infty} k_{n+1} &= -T_L^{-1} \lim_{x \to \infty} T_{N_n} (k_n + h_0) - x_0 + T_L^{-1}(g(a)) \\
&= -T_L^{-1} \lim_{x \to \infty} T_{N_n} \left( \sum_{i=0}^{n} h_i \right) - x_0 + T_L^{-1}(g(a))
\end{aligned}$$

(2.34)

$$\begin{aligned}
k &= -T_L^{-1} \lim_{x \to \infty} \sum_{i=0}^{n} M_i - x_0 + T_L^{-1}(g(a)) \\
&= -T_L^{-1} \sum_{i=0}^{\infty} M_i - x_0 + T_L^{-1}(g(a))
\end{aligned}$$

But by Eqs. (2.21) and (2.15) for p=1, we drive;

$$\sum_{i=0}^{\infty} M_i = T_N\left(\sum_{i=0}^{\infty} h_i\right)$$
(2.35)

So

$$k = -T_L^{-1}T_N\left(\sum_{i=0}^{\infty} h_i\right) - x_0 + T_L^{-1}(g(a))$$

$$k = -T_L^{-1}T_N(k + h_0) - x_0 + T_L^{-1}(g(a))$$

**Lemma 1.** Eq. (2.29) is equivalent to;

$$T_L(x) + T_N(x) - g(a) = 0.$$
(2.36)

**Proof.** We rewrite Eq. (2.29) as follows;

$$k + h_0 = -T_L^{-1}T_N(k + h_0) + T_L^{-1}(g(a))$$
(2.37)

By applying the operator $T_L$ to Eq. (2.36) we derive;

$$T_L(k + h_0) = -T_N(k + h_0) + (g(a))$$

But $x_0 = h_0$, then;

$$T_L(k + h_0) + T_N(k + h_0) = (g(a))$$
(2.38)

By considering $x = k + h_0 = \sum_{n=0}^{\infty} h_i$, Eq. (2.36), has been derived which is the original equation. Then solution of Eq. (2.29) is the same as solution of $P(x) - g(a) = 0$.

## III. Applications

The analytical solution of the model (1.2) using the HPM is

$$S(t) = 1\times10^{-1}e^{-(\mu+\lambda+\nu)t} + \left(1\times10^{-1} - \frac{A_2}{1-\beta} + \frac{A_3}{1-A_4}\right)e^{-t} + \frac{A_2e^{-\beta t}}{1-\beta} - \frac{A_3e^{-A_4 t}}{1-A_4}$$

Here $A_2 = 5.91\times10^{-6}$, $A_3 = 1\times10^{-9}$, $A4 = 6\times10^{-3}$

$$\therefore S(t) = 1\times10^{-1}e^{-6.018t} + 0.09997e^{-t} + 2.955\times10^{-5}e^{-0.8t} - 1\times10^{-9}e^{-0.006t}$$

$$I(t) = 1\times10^{-2}e^{-(\mu+\nu)t} + \left(1\times10^{-2} - \frac{\alpha_1}{1-\alpha_2}\right)e^{-(\mu+\nu)t} + \frac{\alpha_1 e^{-\alpha_2 t}}{1-\alpha_2}$$

Here $\alpha_1 = 1.3\times10\text{-}3$, $\alpha_2 = 6.018$

$$\therefore I(t) = 2\times10^{-2}e^{-6.005t} + 1\times10^{-2}e^{-6.018t}$$

$$D(t) = 2(1\times10^{-3})e^{-(\mu+\gamma)t} + t\nu(1\times10^{-2})e^{-(\mu+\gamma)t}$$

$$\therefore D(t) = (0.002 + 0.06t)e^{-3.005t}$$

$$B(t) = (1\times10^{-4})e^{-\beta t} + \left((1\times10^{-4}) - \psi\right)e^{-\beta t} + \psi e^{-(\mu+\gamma)t}$$

$\Psi = 3.47\times10^{-7}$

$$\therefore B(t) = 1.99653\times10^{-4}e^{-0.8t} + 3.47\times10^{-7}e^{-3.005t}$$

$$R(t) = 1\times10^{-5}e^{-\mu t} + \left(1\times10^{-5} + \xi - \kappa\right)e^{-\mu t} - \xi e^{-(\mu+\gamma)t} + \kappa e^{-\beta t}$$

$\xi = 9.5\times10^{-4}$, $\kappa = -6.29\times10^{-7}$

$$\therefore R(t) = 9.70629\times10^{-4}e^{-0.005t} - 9.5\times10^{-4}e^{-3.005t} - 6.29\times10^{-7}e^{-0.08t}$$

$$A(t) = 1\times10^{-6}e^{-(\phi+\mu)t} + \left((1\times10^{-6}) - \frac{b\omega}{A_4} - \varphi\right)e^{-A_4 t} + \frac{b\omega}{A_4} + \varphi e^{-\alpha_2 t}$$

$\varphi = -35.29$

$$\therefore A(t) = 0.45833 + 34.832e^{-0.006t} - 35.29e^{-6.018t}$$

## IV. Numerical results

Let us consider the values for numerical results are,

$S_0 = 1\times10^{-1}$; $I_0 = 1\times10^{-2}$; $D_0 = 1\times10^{-3}$; $B_0 = 1\times10^{-4}$; $R_0 = 1\times10^{-5}$; $A_0 = 1\times10^{-6}$.

$\mu = 0.005$, $\lambda = 0.013$, $v = 6$, $\beta = 0.8$, $\omega = 0.055$, $\sigma = 0.7$, $\delta = 0.005$, $b = 0.05$, $\phi = 0.001$, $\gamma = 3$, $\rho = 0.05$

Let us use MatLab software to obtain the sixth-order expansions for S(t), I(t), D(t), B(t), R(t) and A(t):

$S(t) = 0.1 + 0.01595ht + 0.001798h^2t + 0.00278h^3t + 0.0001487h^4t + 0.0003585h^5t + 0.005785h^6t + 0.0002486h^2t^2 + 0.0004324h^3t^2 + 0.006788h^4t^2 + 0.083721h^5t^2 + 0.0094426h^6t^2 + 0.0008022h^3t^3 + 0.0000037878h^4t^3 + 0.076555656h^5t^3 + 0.0056488h^6t^3 \ldots$ (4.1)

$I(t) = 0.01 + 0.0014855ht + 0.0045568h^2t + 0.0005568h^3t + 0.000854846h^4t + 0.000045544532h^5t + 0.0005496 + h^6t + 0.0004668h^2t^2 + 0.0000545155h^3t^2 + 0.00055669h^4t^2 + 0.004047956h^5t^2 + 0.000054876599h^6t^2 + 0.000045956531h^3t^3 + 0.000098462h^4t^3 + 0.00005857656h^5t^3 + 0.00054167989h^6t^3 \ldots$ (4.2)

$D(t) = 0.001+ 0.0006555ht + 0.0004862h^2t + 0.00015463h^3t + 0.00046387h^4t + 0.00065452h^5t + 0.00005697+h^6t + 0.000077956h^2t^2 + 0.0006713h^3t^2 + 0.000338425h^4t^2 + 0.0008952h^5t^2 + 0.00075199h^6t^2 + 0.00097123h^3t^3 + 0.00003541h^4t^3+ 0.00087661h^5t^3+ 0.0008553h^6t^3 \ldots$ (4.3)

$B(t) = 0.0001+ 0.00005456ht + 0.00005469h^2t + 0.000056545h^3t + 0.00003221h^4t + 0.00001546h^5t + 0.000067556+h^6t + 0.00008165h^2t^2 + 0.00009482h^3t^2 + 0.00000697h^4t^2 + 0.000089945h^5t^2 + 0.00009561h^6t^2 + 0.0000248968h^3t^3 + 0.00003786h^4t^3+ 0.00007665h^5t^3+ 0.000086551h^6t^3 \ldots$

(4.4)

$R(t) = 0.00001+ 0.00000545ht + 0.000005853h^2t + 0.000007626h^3t + 0.000007543h^4t + 0.000007236h^5t + 0.000006512+h^6t + 0.0000076565h^2t^2 + 0.000009225h^3t^2 + 0.0000006481h^4t^2 + 0.000007551h^5t^2 + 0.000009125h^6t^2 + 0.000002756h^3t^3 + 0.000003912h^4t^3+ 0.00000794h^5t^3+ 0.000004665h^6t^3 \ldots$

(4.5)

$A(t) = 0.000001+ 0.0000003551ht + 0.000000466h^2t + 0.000000658h^3t + 0.0000008484h^4t + 0.0000008493h^5t + 0.0000007555+h^6t + 0.0000007561h^2t^2 + 0.00000031124h^3t^2 + 0.000000076551h^4t^2 + 0.00000007552h^5t^2 + 0.00000004565h^6t^2 + 0.0000000452h^3t^3 + 0.0000000465h^4t^3+ 0.0000005545h^5t^3+ 0.00000078954h^6t^3 \ldots$

(4.6)

## V. Discussion

The spread of HBV is very difficult procedure. It is controlled and subjective by many relating aspects. We have examined the sensitive, idle, dangerous,bearer, reacquire and antibiotic individuals, and also individuals who are carrying the diseases before treatment and after treatment are shown through parameter values. Figure 2 shows that, when the force of infection increases, the sensitive population got decreased therefore HBV also would get increased. When the mortality rate and transmission coefficients are increased then the sensitive populations are decreased. This can be shown in figures 3, 4 and 5. These are only depended age and time.

**FIGURE 2**The changes in sensitive populations force of HBV infection

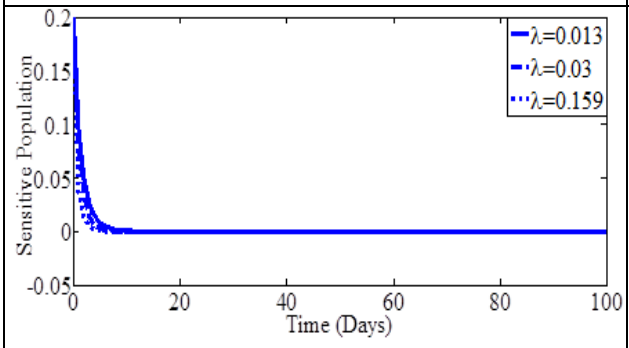



**FIGURE 3** The changes in sensitive populations of v

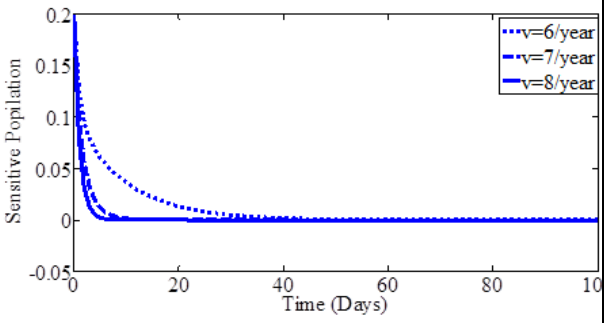



**FIGURE 4** The changes in sensitive populations of the mortality rate

**FIGURE 5** The changes in sensitive populations of transmission coefficient

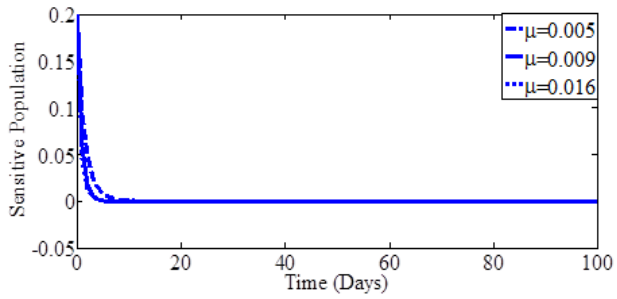



**FIGURE 6** The changes in idle populations of natural mortality

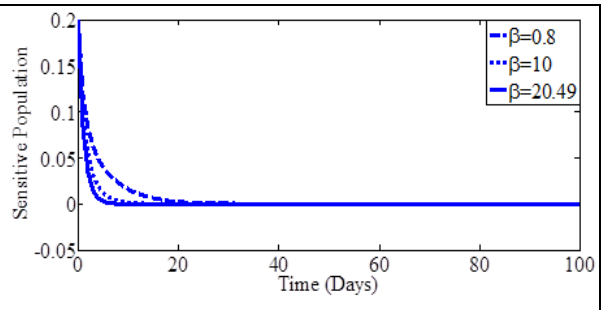
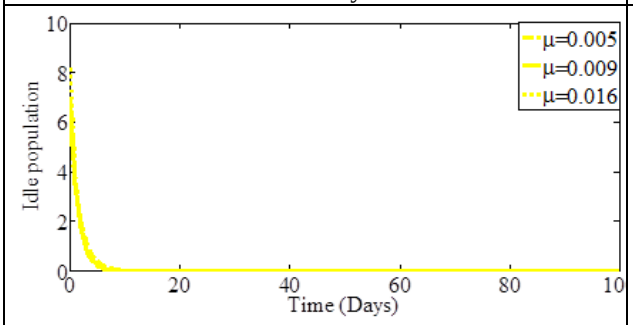
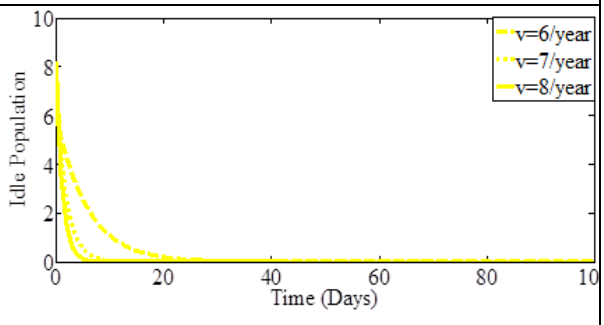



**FIGURE 7**The changes in idle populations of individual leave the idle class




The idle population is decreased when the natural mortality rate and the rate of individuals leave from idle class are increased. This can be shown in figures 7 and 8. The infectious condition of dangerous class will be decreased when the natural mortality and v values are increased. This can be shown in figures 9 and 10. Finally the individuals leave from the dangerous class which is shown in figure 11.

**FIGURE 8** The changes in dangerous populations of natural mortality rate

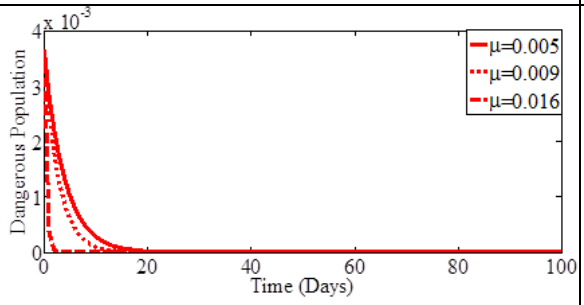



**FIGURE 9** The changes in dangerous populations of individual leave the idle class

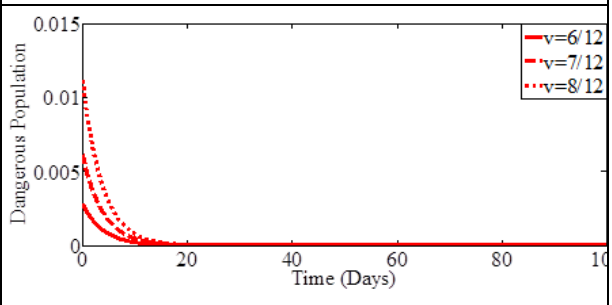



**FIGURE 10** The changes of populations leave the dangerous class

**FIGURE 11** The changes in bearer populations of natural mortality rate

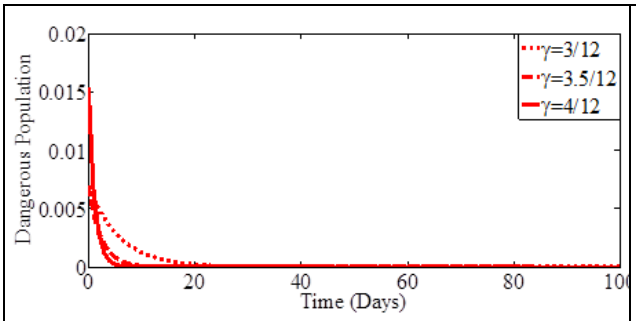



**FIGURE 12** The changes in bearer populations of β

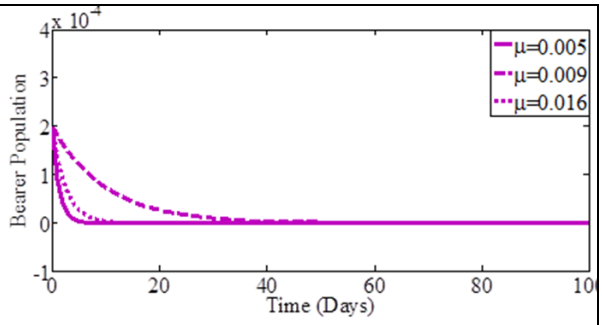
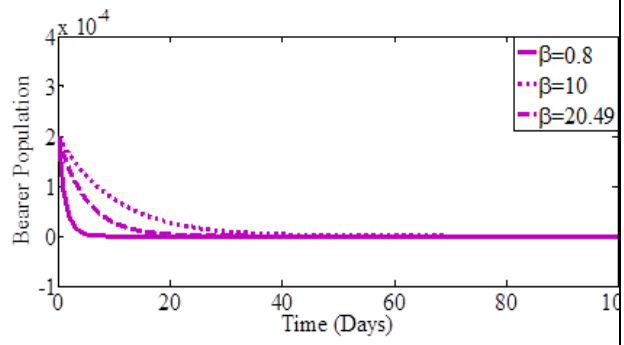
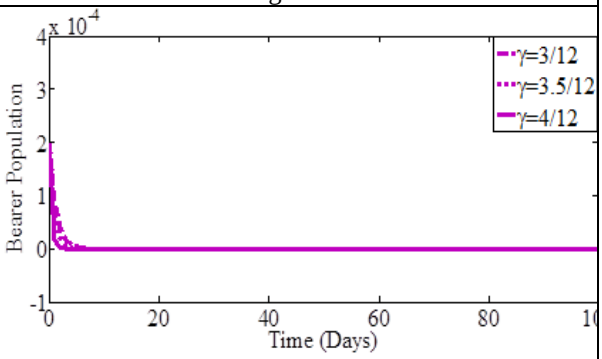



**FIGURE 13** The changes in bearer populations leave the dangerous class

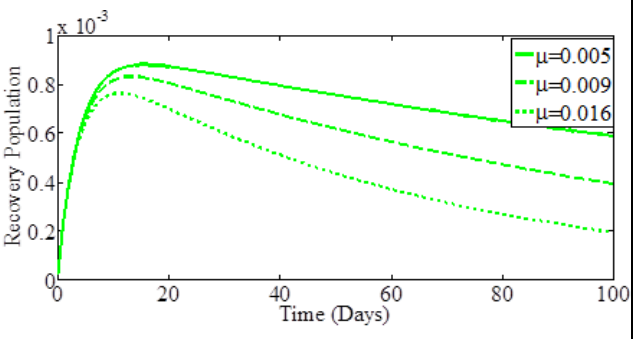



When the continuous treatment is given the impact of HBV infection decreased compared with dangerous class. This can be shown in figures 12, 13 and 14. This process is continued till the patient will be reacquired. The figures 15, 16 and 17 are shown the reacquire population is increased when the continuous treatment is given. After recovering, some patients are advised to take antibiotic. Because of that protection after HBV reacquire keep up the lifetime of the population, though the protection of vaccination may vanish over time.

**FIGURE 14** The changes in reacquire populations of natural mortality rate

**FIGURE 15** The changes in reacquire populations of transmission coefficient



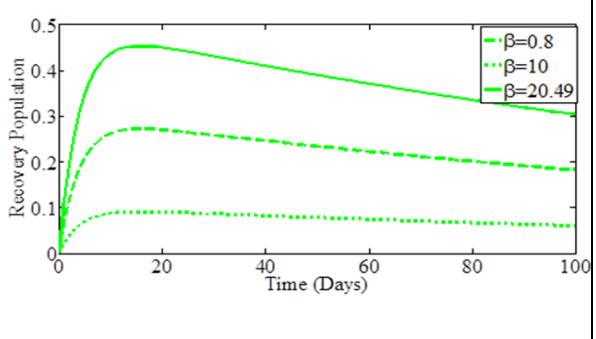




**FIGURE 16** The changes in reacquire populations of leaving dangerous class

**FIGURE 17** The changes in antibiotic populations of natural mortality rate

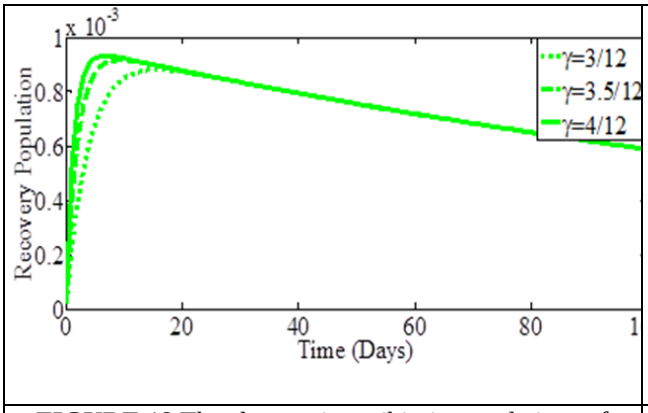



**FIGURE 18** The changes in antibiotic populations of births with successful vaccination

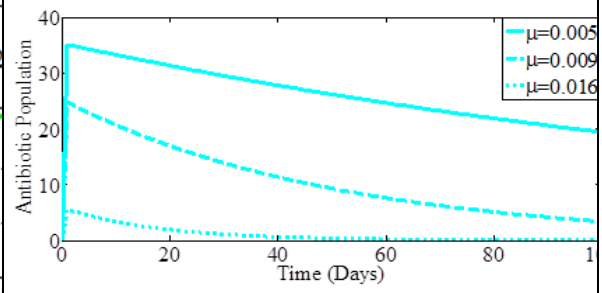
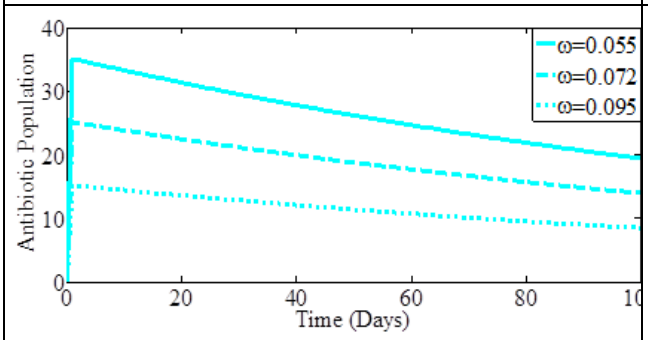
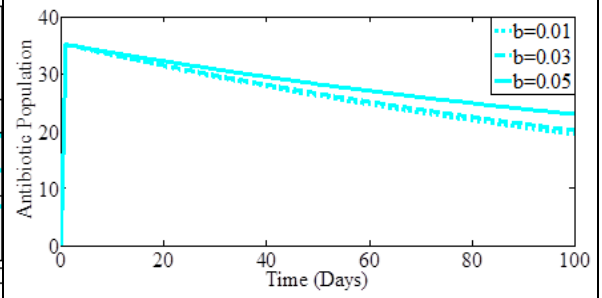



**FIGURE 19** The changes in antibiotic populations of birth rate.




## VI. Conclusion

The transmission dynamicsmathematical model of HBV is individual from the macroscopic observation to put on the spread of hepatitis B in the population. Age-factor is the greatest significant characteristics in the transmission of HBV. From this mathematical model it is redirected the spread of HBV and its consequences which control the transmission of HBV. The numerical simulation provides numerical understanding of the transmission of HBV that results and makes thebesttreatment for individual patients.The numerical simulations obtained up to six order approximations with the help of Matlab software. We concluded that our work take minimum number of days to cure HBV. This research paper can be framework for the young researchers to do a further research.

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